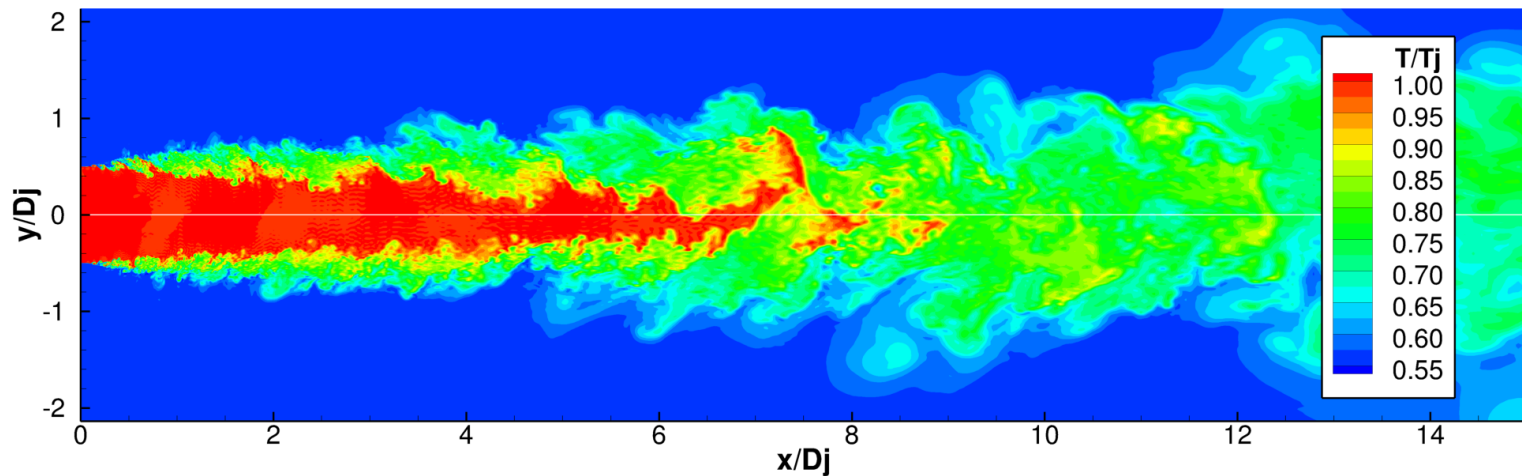


Prediction of Turbulent Temperature Fluctuations in Hot Jets



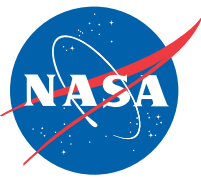
Jim DeBonis

Inlets and Nozzles Branch

NASA Glenn Research Center

Cleveland, Ohio

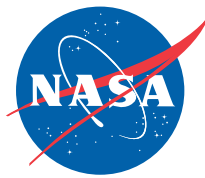
james.r.debonis@nasa.gov



Prediction of Heated Jets

- Jets with significant temperature differences have many important applications
 - Aeroacoustics
 - Cooling flows
 - Fuel injectors
 - IR signatures
- Standard CFD methods (RANS) do a very poor job predicting these flows
- Possible reasons
 - Turbulence model
 - Turbulent Prandtl number variation
 - Turbulent heat flux modeling

Reynolds-Averaged Navier-Stokes



$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i} (\bar{\rho} \hat{u}_i) = 0$$

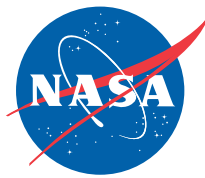
$$\frac{\partial}{\partial t} (\bar{\rho} \hat{u}_i) + \frac{\partial}{\partial x_j} (\bar{\rho} \hat{u}_i \hat{u}_j) + \frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \bar{\tau}_{ij}}{\partial x_j} + \frac{\partial}{\partial x_j} (\overline{\rho u'_i u'_j}) = 0$$

$$\frac{\partial}{\partial t} (\bar{\rho} \hat{e}_t) + \frac{\partial}{\partial x_j} (\bar{\rho} \hat{u}_j \hat{e}_t + \hat{u}_j \bar{p}) - \frac{\partial}{\partial x_j} [\hat{u}_i \bar{\tau}_{ij} - \hat{u}_i (\overline{\rho u'_i u'_j})] + \frac{\partial}{\partial x_j} (\bar{q}_j + c_p \overline{\rho u'_j T'}) = 0$$

$$\bar{\tau}_{ij} = 2\mu \left[\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{1}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right]$$

$$\bar{q}_j = -c_p \frac{\mu}{Pr} \frac{\partial T}{\partial x_j}$$

Reynolds-Averaged Navier-Stokes



$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i} (\bar{\rho} \hat{u}_i) = 0$$

Reynolds stress

$$\frac{\partial}{\partial t} (\bar{\rho} \hat{u}_i) + \frac{\partial}{\partial x_j} (\bar{\rho} \hat{u}_i \hat{u}_j) + \frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \bar{\tau}_{ij}}{\partial x_j} + \frac{\partial}{\partial x_j} \boxed{\overline{\rho u'_i u'_j}} = 0$$

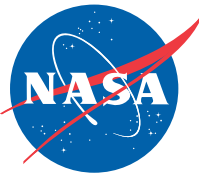
$$\frac{\partial}{\partial t} (\bar{\rho} \hat{e}_t) + \frac{\partial}{\partial x_j} (\bar{\rho} \hat{u}_j \hat{e}_t + \hat{u}_j \bar{p}) - \frac{\partial}{\partial x_j} \left[\hat{u}_i \bar{\tau}_{ij} - \hat{u}_i \boxed{\overline{\rho u'_i u'_j}} \right] + \frac{\partial}{\partial x_j} \left(\bar{q}_j + \boxed{c_p \overline{\rho u'_j T'}} \right) = 0$$

$$\bar{\tau}_{ij} = 2\mu \left[\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{1}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right]$$

$$\bar{q}_j = -c_p \frac{\mu}{Pr} \frac{\partial T}{\partial x_j}$$

Turbulent heat flux

Turbulence Closure



- Reynolds stress
 - Boussinesq approximation

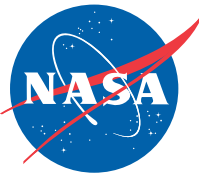
$$-\overline{\rho u'_i u'_j} = \mu_t \left(2\hat{S}_{ij} - \frac{2}{3} \frac{\partial \hat{u}_k}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \bar{\rho} k \delta_{ij}$$

$$\hat{S}_{ij} = \frac{1}{2} \left(\frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i} \right)$$

- Turbulent heat flux
 - Gradient diffusion & Reynolds Analogy

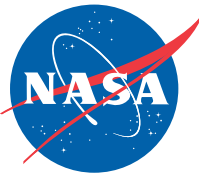
$$c_p \overline{\rho u'_j T'} = -c_p \frac{\mu_t}{Pr_t} \frac{\partial T}{\partial x_j}$$

Approach



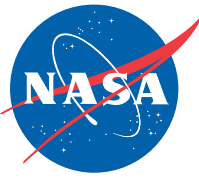
- Use large-eddy simulation (LES) to examine the turbulent heat flux vector, q^T_i , and turbulent Prandtl number, Pr_t , in hot jets
- Validate the LES using experimental data
 - PIV data for velocity
 - Rayleigh scattering and Raman spectroscopy for temperature
- Make a leap of faith that if u' and T' are validated separately, then $u'T'$ should not be too bad
- Compare results to Reynolds-Averaged Navier-Stokes (RANS) simulations and evaluate

Large-Eddy Simulations



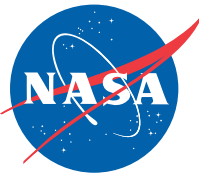
- WRLES code
 - Explicit high-resolution finite-difference code
 - 11-pt DRP differencing scheme (Bogey & Bailly, 2004) with matching filter
 - 4-stage, 3rd order Runge-Kutta time stepping
 - Hybrid MPI/OpenMP parallelization
- Grid
 - Structured grid
 - 36 million points
 - 912x184x181 points downstream of nozzle exit

RANS Simulations



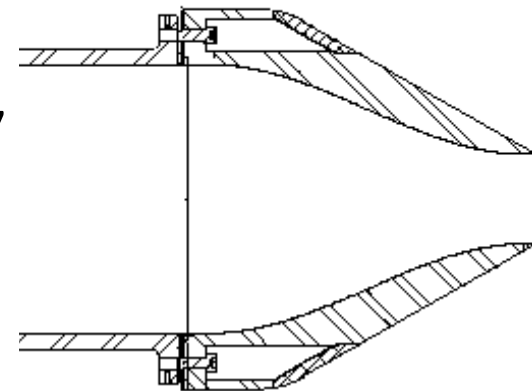
- Wind-US
 - Finite-volume
 - Structured grid axisymmetric mode
 - 2nd-order upwind biased RHS
 - Full block-implicit LHS
 - SST-V turbulence model (vorticity based production term)
- Grid
 - Taken from turbmodels.larc.nasa.gov
 - 73,151 points
 - Downstream of the nozzle exit 257x251 points
 - Provides grid converged solutions with Wind-US

Round Jet Experiments

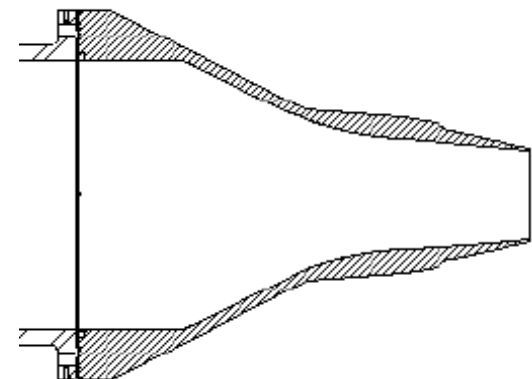


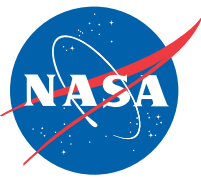
- Small Hot Jet Acoustic Rig (SHJAR)
- 2-inch nozzles: ARN2 and SMC000
- PIV Velocity Data
 - Bridges and Wernet, NASA TM 2011- 216807
 - Consensus dataset
 - Verified against hotwire and LDV
- Rayleigh Scattering Temperature Data
 - Mielke et al, AIAA Journal, Vol. 47, No. 4, 2009
 - Point measurement
- Raman Spectroscopy Temperature Data
 - Locke and Wernet, NASA TM 2017-219504
 - Point measurement

ARN2



SMC000





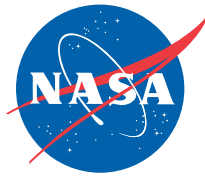
LES Methodology

- Implicit LES
- Nozzle boundary layer
 - No attempt to resolve a turbulent boundary layer
 - Transition occurs quickly in mixing layer
- Non-dimensional time
$$t^* = \frac{tD_j}{U_j}$$
- Startup time: $60t^*$
- Averaging time: $> 180t^*$

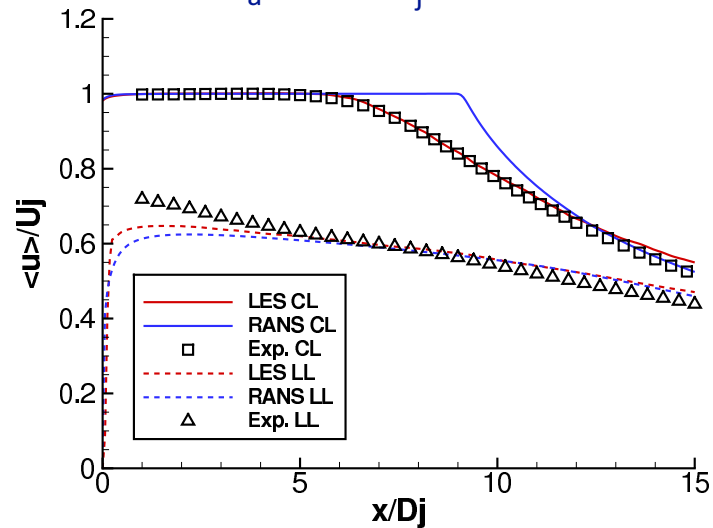
Flow Conditions

Set Point	M_a	T_j/T_∞	NPR	M_j
3	0.5	0.950	1.197	0.513
23	0.5	1.764	1.102	0.376
27	0.9	1.764	1.357	0.678

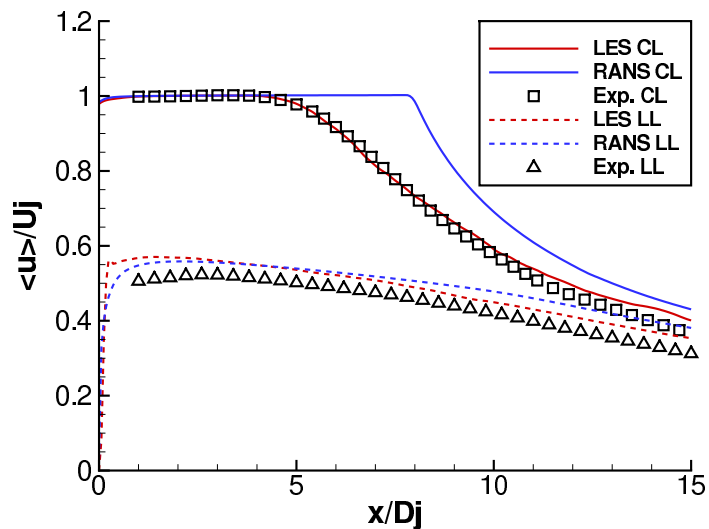
Mean Velocity



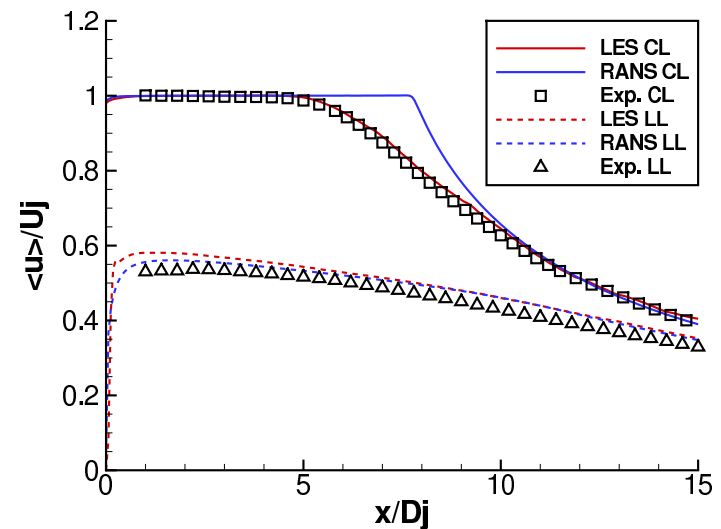
SP 3: $M_a = 0.5$, $T_j/T_\infty = 0.950$



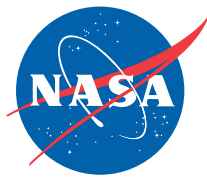
SP 23: $M_a = 0.5$, $T_j/T_\infty = 1.764$



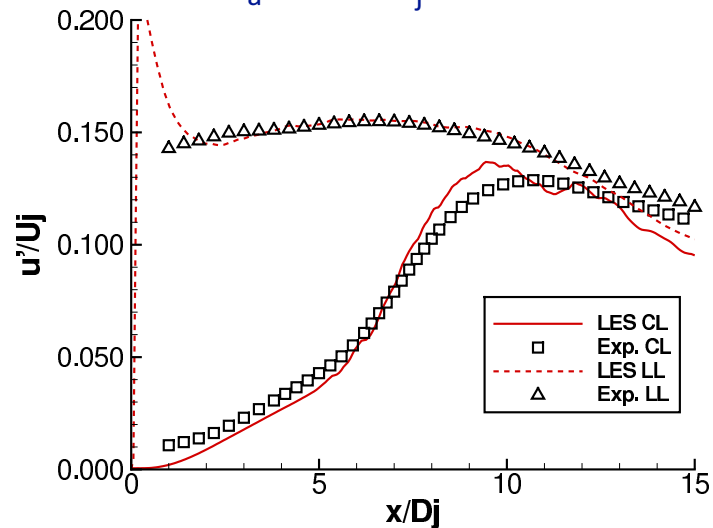
SP 27: $M_a = 0.9$, $T_j/T_\infty = 1.764$



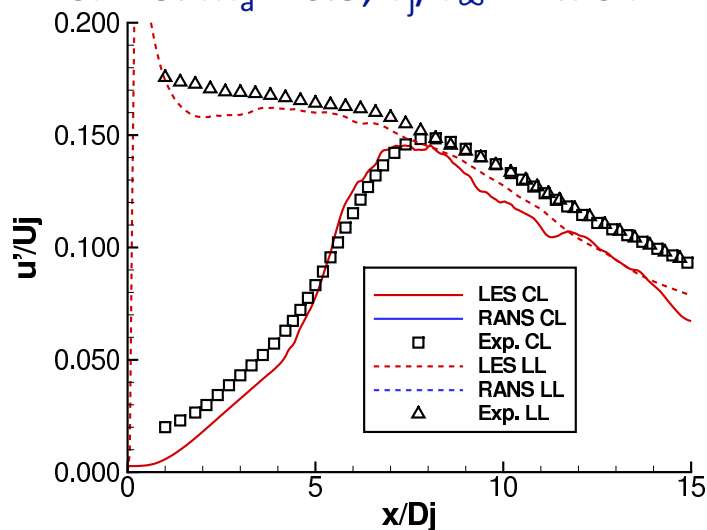
Axial Turbulence Intensity



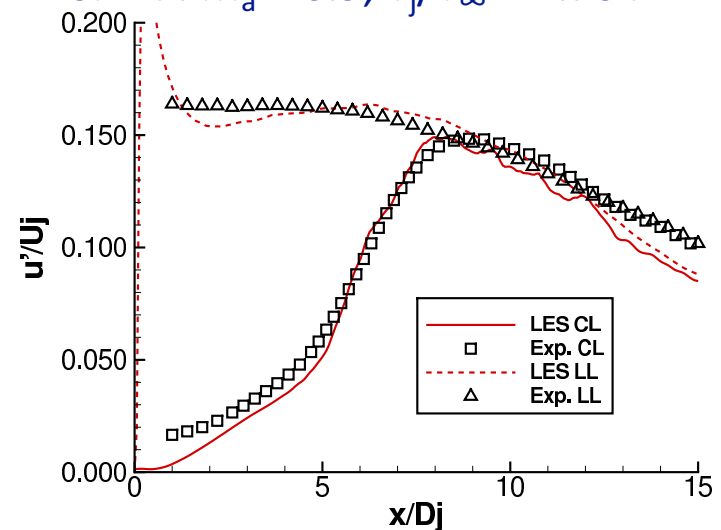
SP 3: $M_a = 0.5$, $T_j/T_\infty = 0.950$



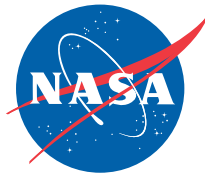
SP 23: $M_a = 0.5$, $T_j/T_\infty = 1.764$



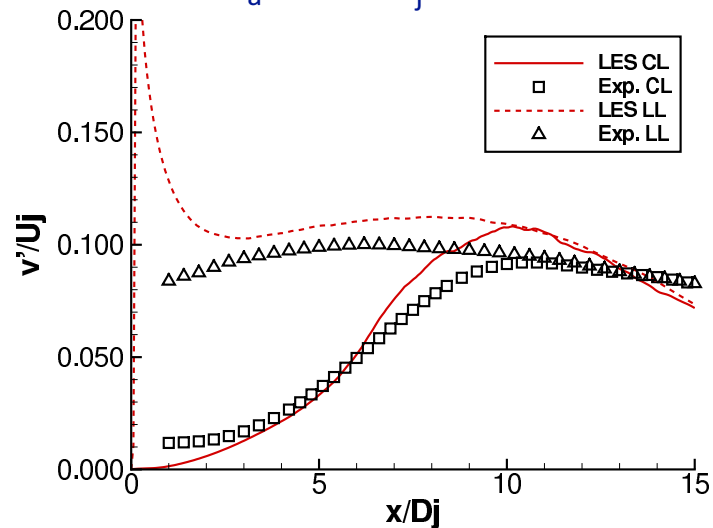
SP 27: $M_a = 0.9$, $T_j/T_\infty = 1.764$



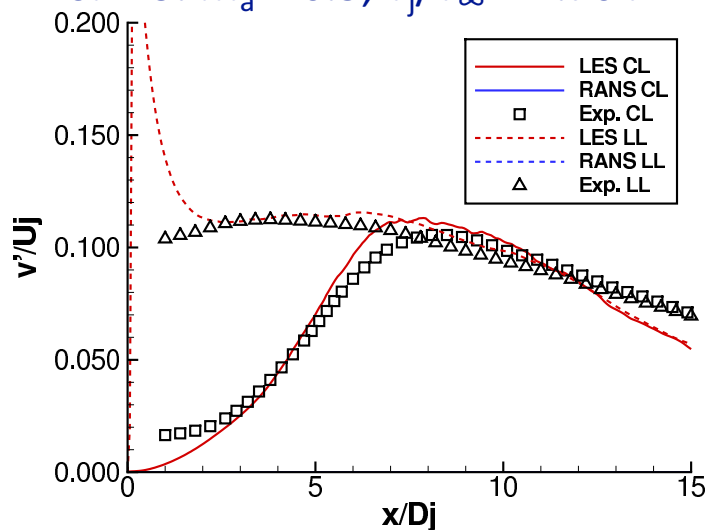
Radial Turbulence Intensity



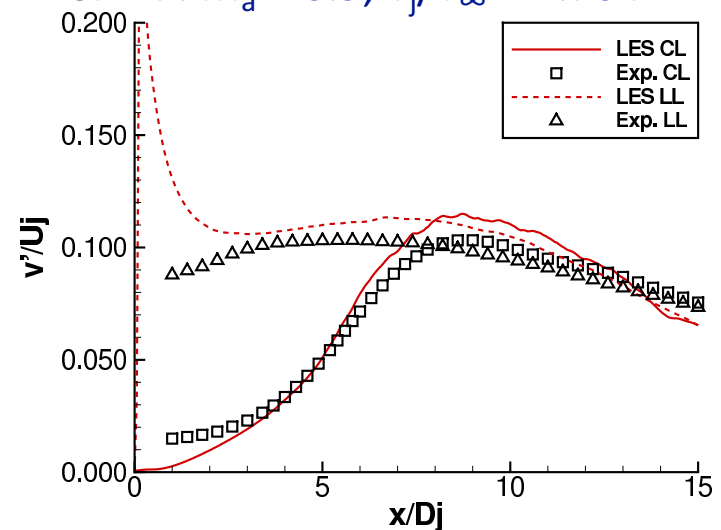
SP 3: $M_a = 0.5$, $T_j/T_\infty = 0.950$



SP 23: $M_a = 0.5$, $T_j/T_\infty = 1.764$

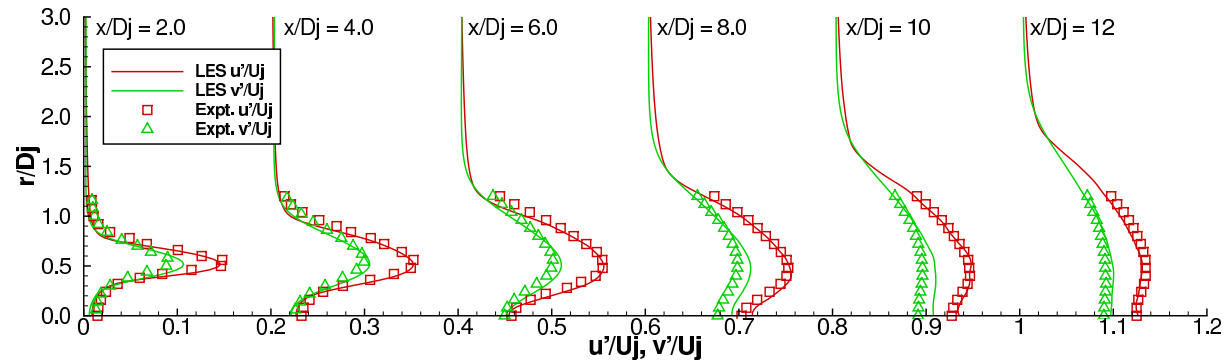


SP 27: $M_a = 0.9$, $T_j/T_\infty = 1.764$

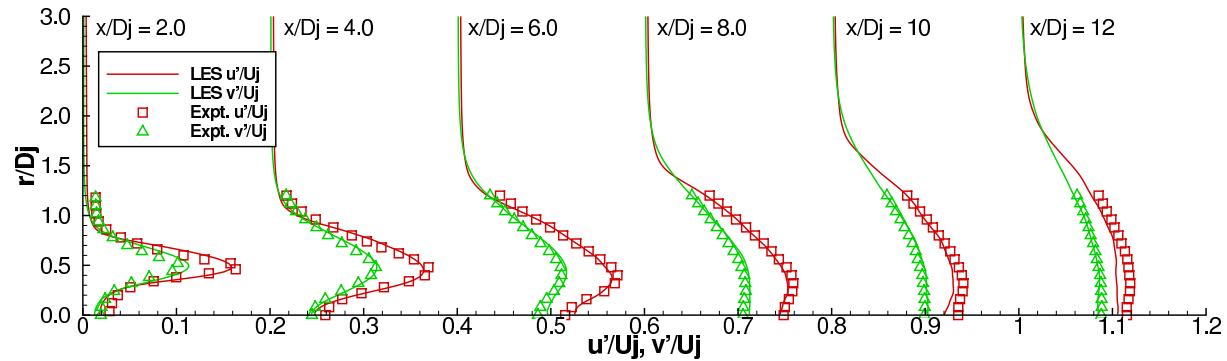


Radial Profiles – u' & v'

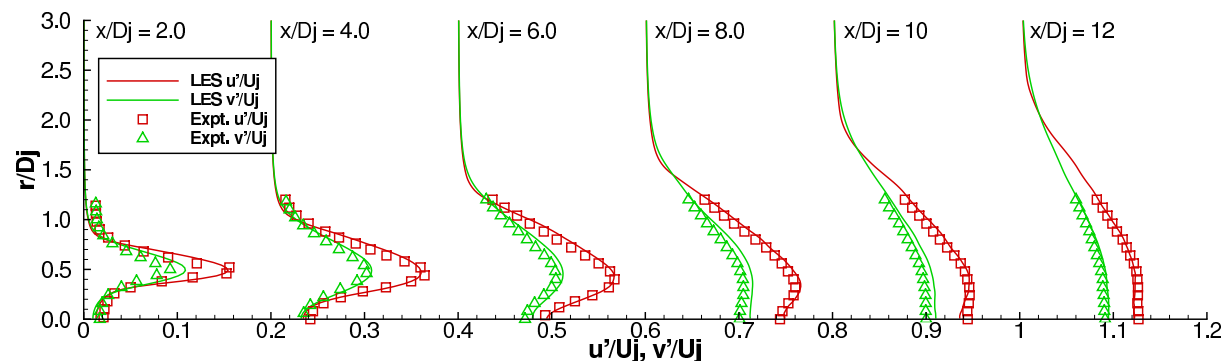
SP 3
 $M_a = 0.5$
 $T_j/T_\infty = 0.950$



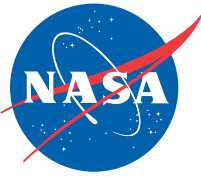
SP 23
 $M_a = 0.5$
 $T_j/T_\infty = 1.764$



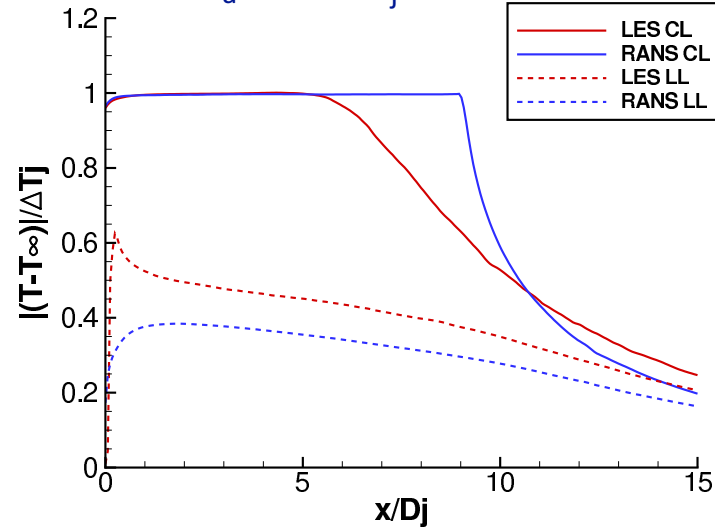
SP 27
 $M_a = 0.9$
 $T_j/T_\infty = 1.764$



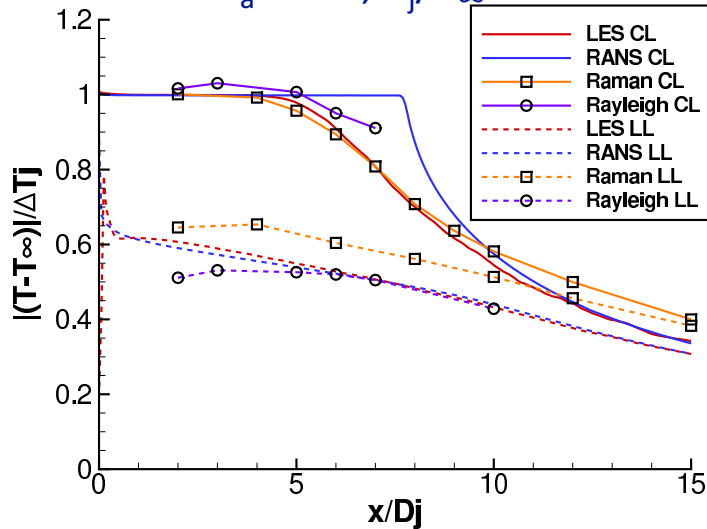
Mean Temperature



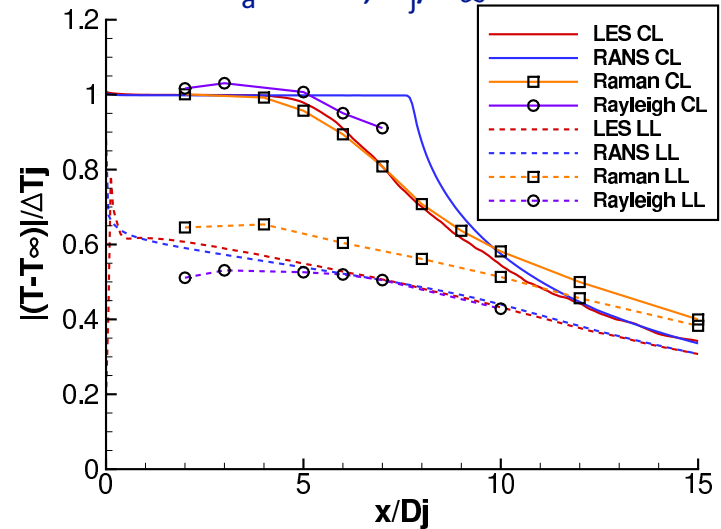
SP 3: $M_a = 0.5$, $T_j/T_\infty = 0.950$



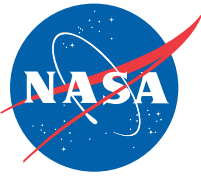
SP 23: $M_a = 0.5$, $T_j/T_\infty = 1.764$



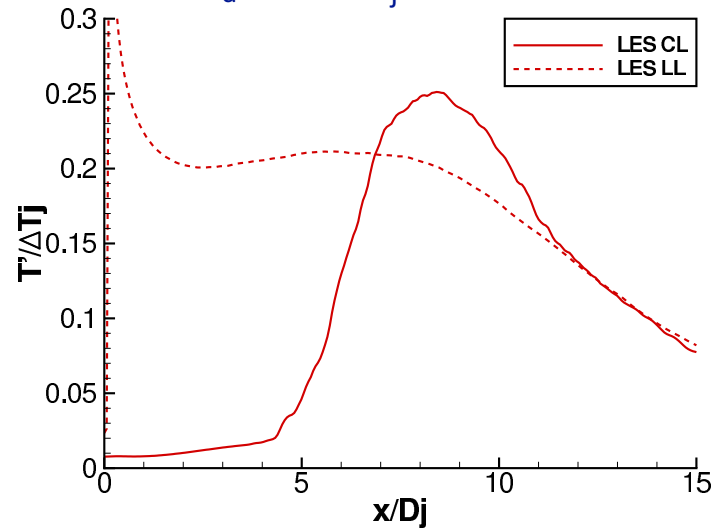
SP 27: $M_a = 0.9$, $T_j/T_\infty = 1.764$



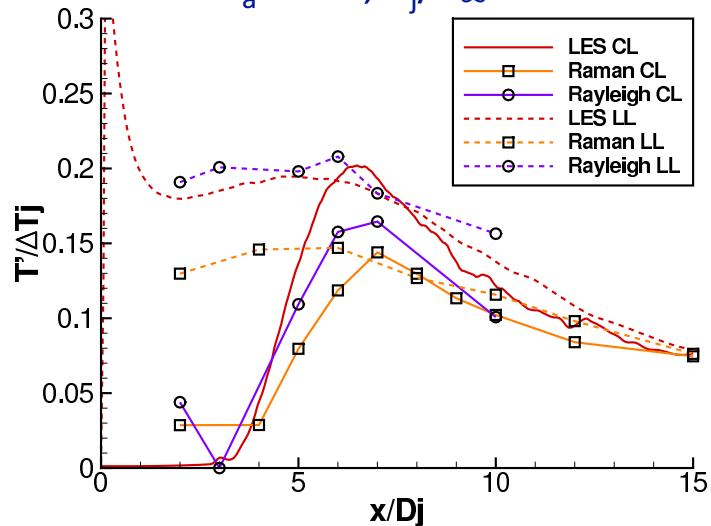
RMS Temperature



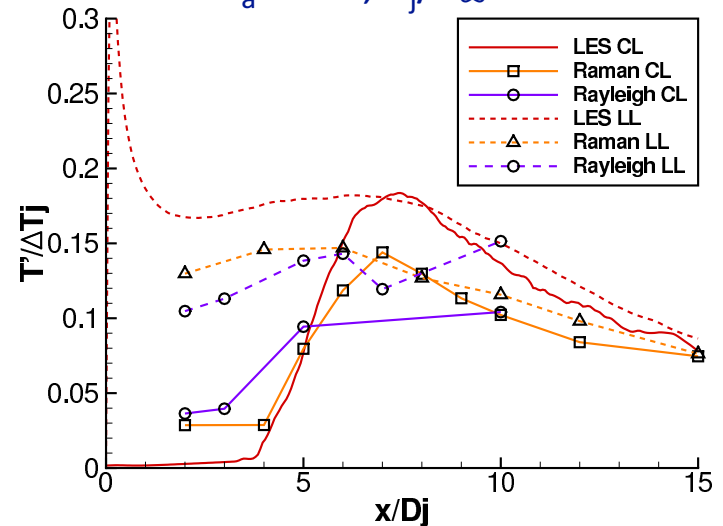
SP 3: $M_a = 0.5$, $T_j/T_\infty = 0.950$



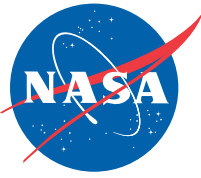
SP 23: $M_a = 0.5$, $T_j/T_\infty = 1.764$



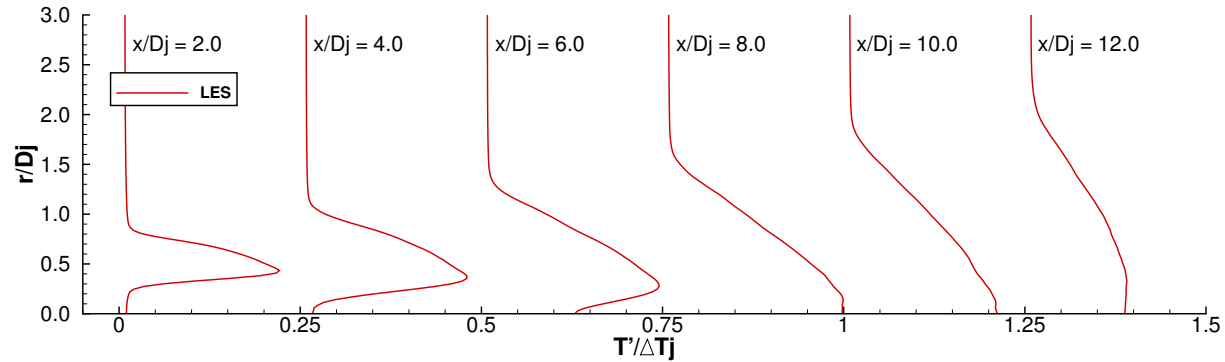
SP 27: $M_a = 0.9$, $T_j/T_\infty = 1.764$



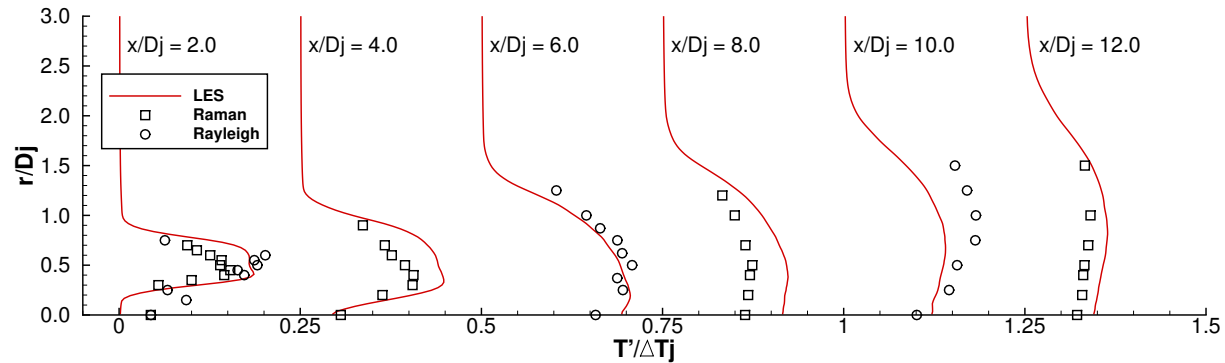
Radial Profiles – T'



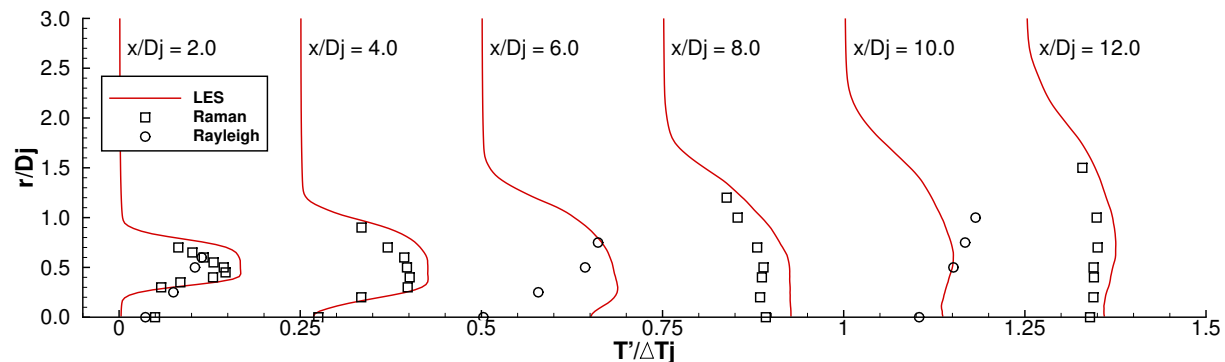
SP 3
 $M_a = 0.5$
 $T_j/T_\infty = 0.950$



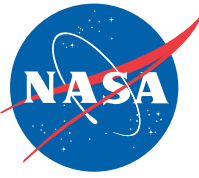
SP 23
 $M_a = 0.5$
 $T_j/T_\infty = 1.764$



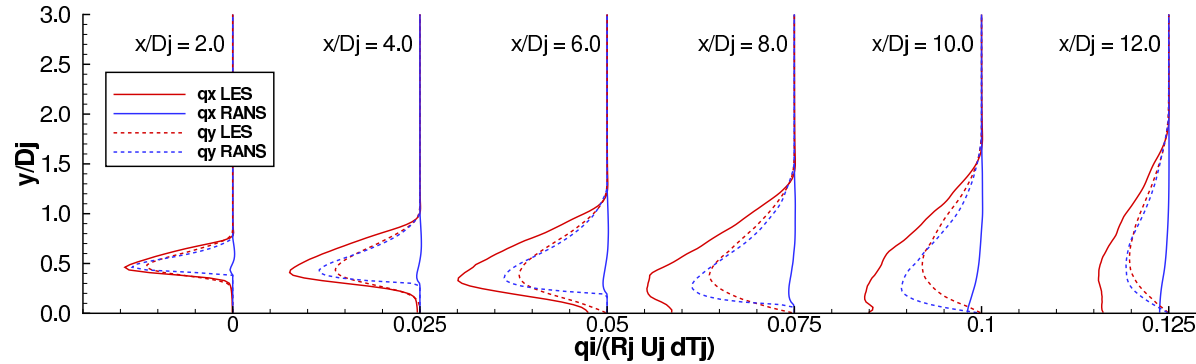
SP 27
 $M_a = 0.9$
 $T_j/T_\infty = 1.764$



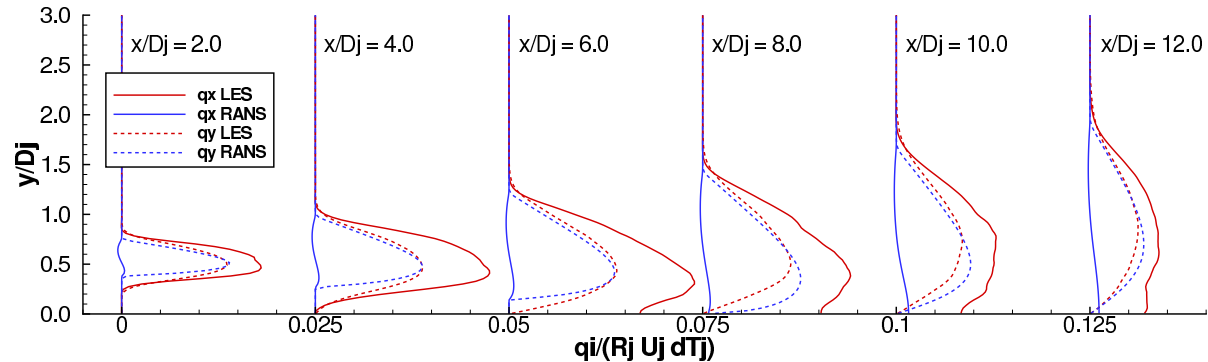
Radial Profiles – q_x^T , q_y^T



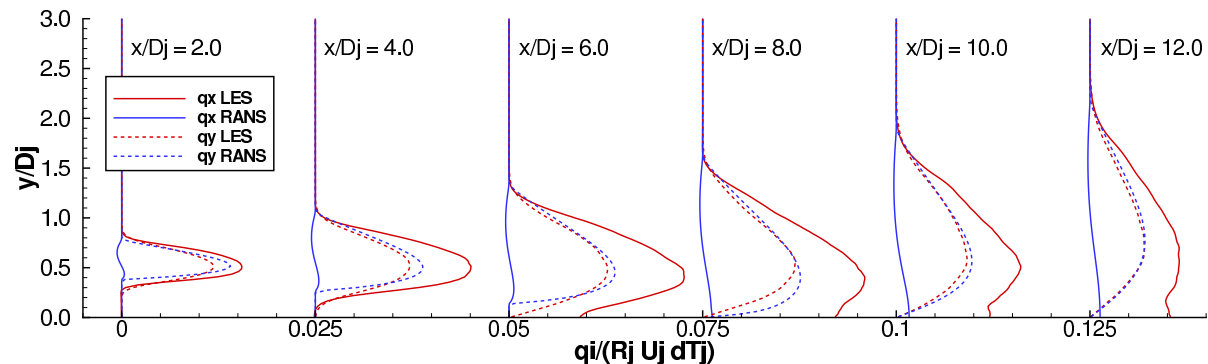
SP 3
 $M_a = 0.5$
 $T_j/T_\infty = 0.950$



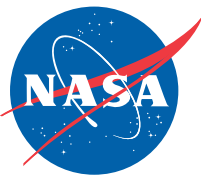
SP 23
 $M_a = 0.5$
 $T_j/T_\infty = 1.764$



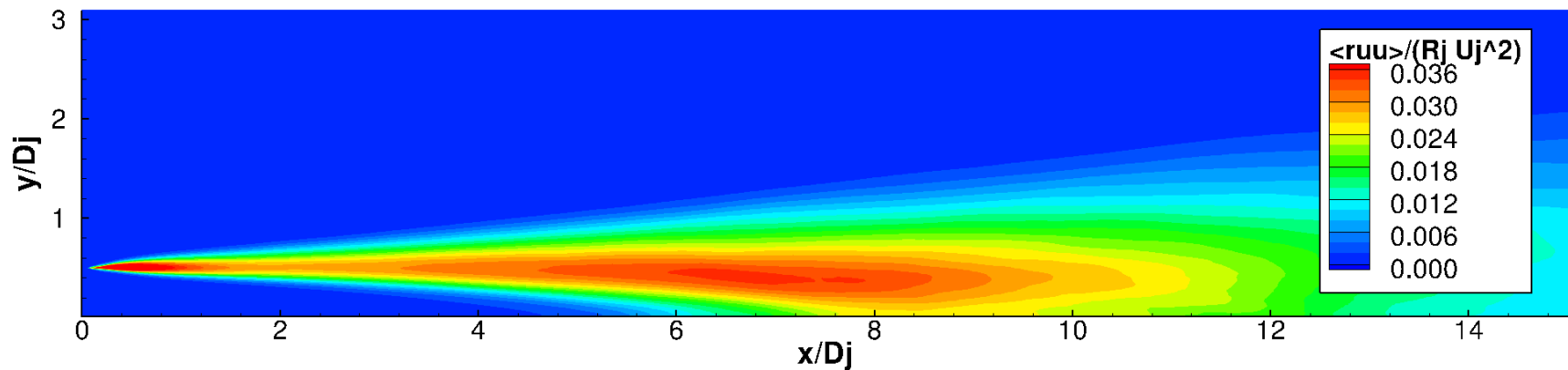
SP 27
 $M_a = 0.9$
 $T_j/T_\infty = 1.764$



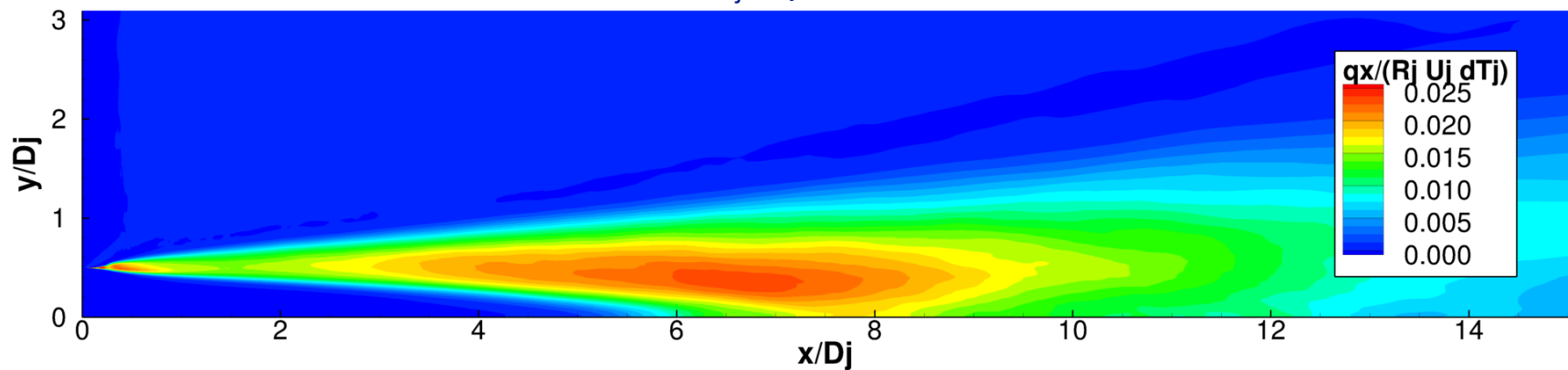
Contours of $\langle \rho u u \rangle$ and q_x



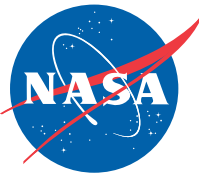
$\langle \rho u u \rangle$



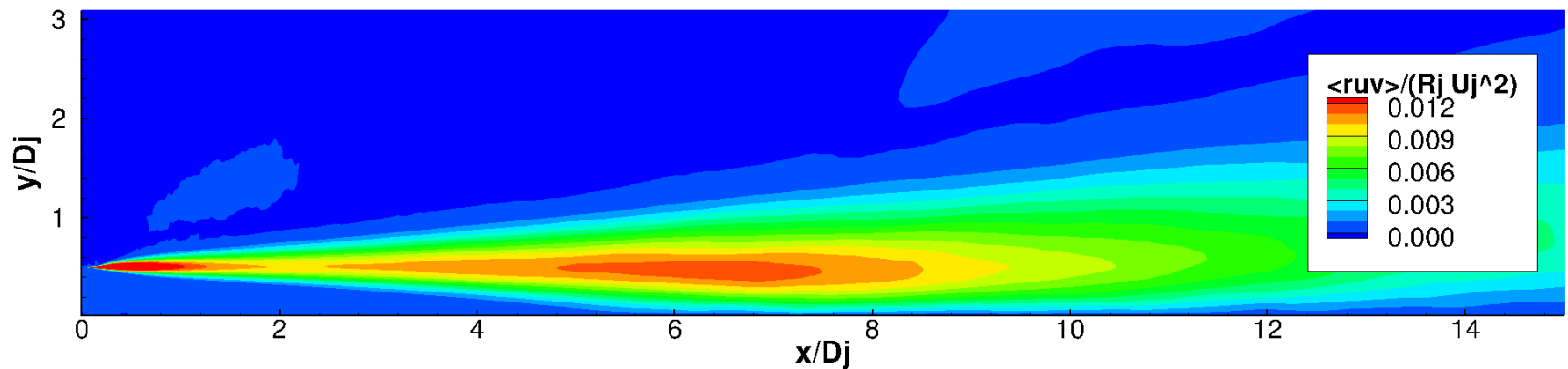
$q_x / (\rho U_j \Delta T_j) = \langle \rho u T \rangle$



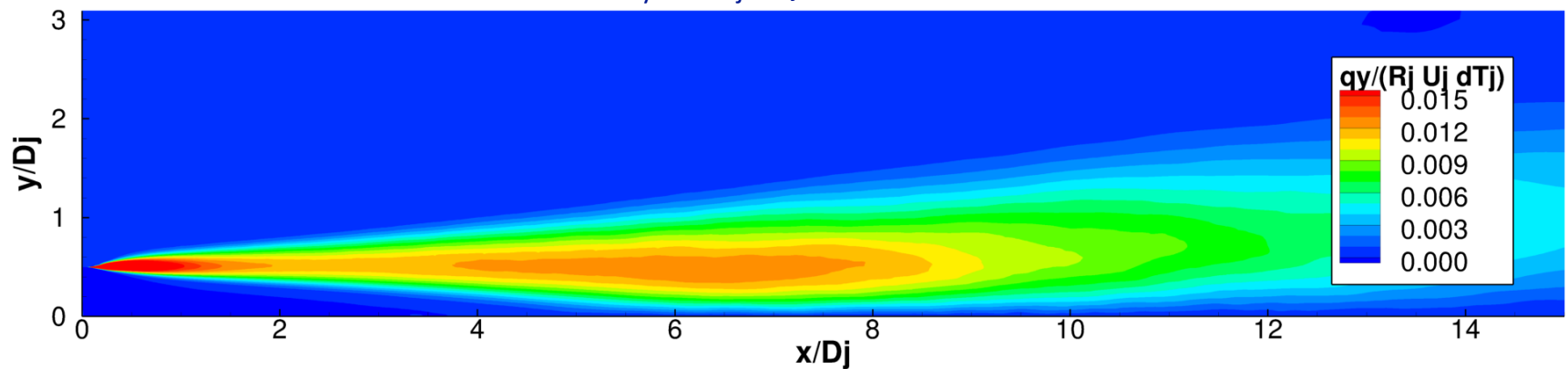
Contours of $\langle \rho uv \rangle$ and q_y



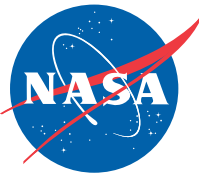
$\langle \rho uv \rangle$



$q_y / (\rho U_j \Delta T_j) = \langle \rho v T \rangle$



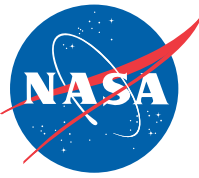
Turbulent Heat Flux



- Turbulent heat flux model

$$\overline{c_p \rho u'_j T'} = -c_p \frac{\mu_t}{Pr_t} \frac{\partial T}{\partial x_j}$$

- Radial component
 - RANS & LES agree surprising well
 - Mean temperature gradient is in radial direction
- Axial component
 - LES predicts heat flux larger than radial component
 - RANS model predicts almost no heat flux
 - No temperature gradient in this direction
- LES heat flux agrees with experiments in the literature
 - Magnitude
 - Fabris (1979): $\langle uT \rangle$ & $\langle vT \rangle$ similar in magnitude
 - Tavoularis & Corrsin (1981): $\langle uT \rangle$ larger than $\langle vT \rangle$
 - Alignment (angle between temp. gradient and heat flux vector)
 - Current LES: 57°
 - Tavoularis & Corrsin (1981): 63°
- Gradient diffusion model is not appropriate for this flow
- Heat flux behavior is analogous to momentum flux



Effect on the Energy Equation

- Energy Equation

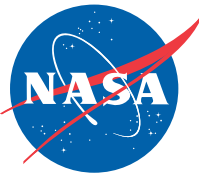
$$\frac{\partial}{\partial t} (\bar{\rho} \hat{e}_t) + \frac{\partial}{\partial x_j} (\bar{\rho} \hat{u}_j \hat{e}_t + \hat{u}_j \bar{p}) - \frac{\partial}{\partial x_j} [\hat{u}_i \bar{\tau}_{ij} - \hat{u}_i (\overline{\rho u'_i u'_j})] + \frac{\partial}{\partial x_j} (\bar{q}_j + c_p \overline{\rho u'_j T'}) = 0$$

Divergence!

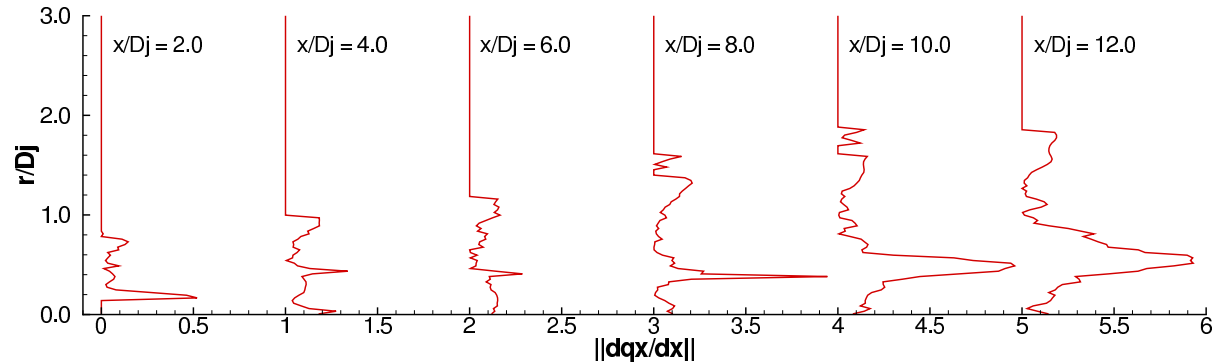
- Quantify the contribution of the missing axial component

$$\left\| \frac{\partial q_x^T}{\partial x} \right\| = \frac{\left| \frac{\partial q_x^T}{\partial x} \right|}{\sqrt{\left(\frac{\partial q_x^T}{\partial x} \right)^2 + \left(\frac{\partial q_r^T}{\partial r} \right)^2}}$$

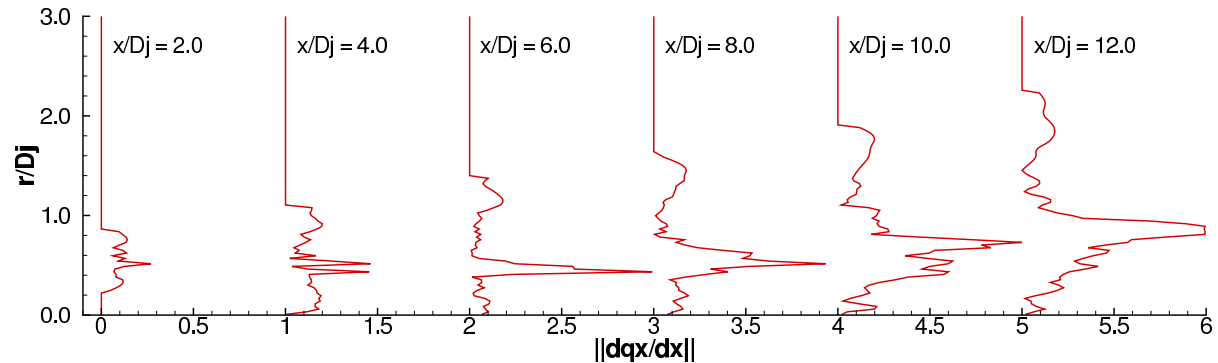
Radial Profiles – $\left\| \frac{\partial q_x^T}{\partial x} \right\|$



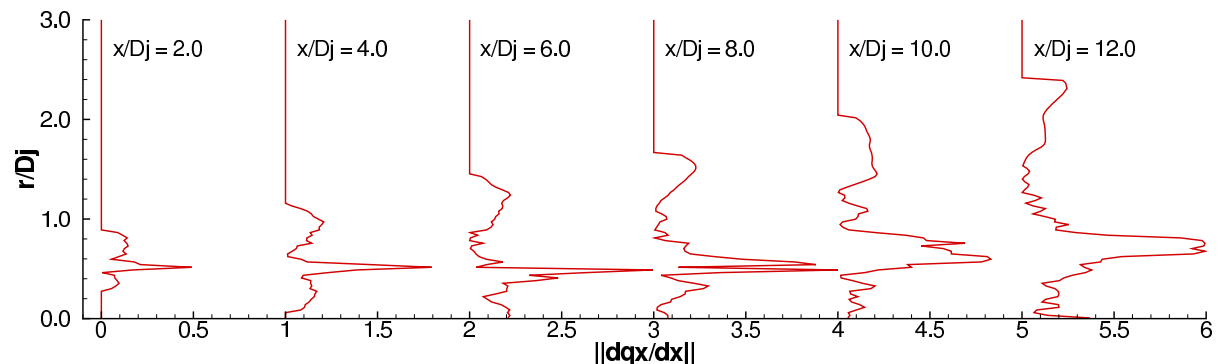
SP 3
 $M_a = 0.5$
 $T_j/T_\infty = 0.950$

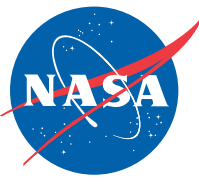


SP 23
 $M_a = 0.5$
 $T_j/T_\infty = 1.764$



SP 27
 $M_a = 0.9$
 $T_j/T_\infty = 1.764$





Turbulent Prandtl Number

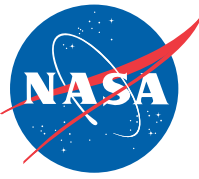
- Treated as a constant but varies, $0.5 < Pr_t < 1.0$
- $Pr_t = 0.7$ is standard value for jets
- Variable Pr_t models often cited as a solution to these types of problems
- Yoder's (2016) recent results showed no advantage for jets
- Can be computed from the LES

$$\epsilon_m = -\frac{\overline{\rho u'v'}}{\overline{\rho} \frac{\partial \overline{u}}{\partial y}}$$

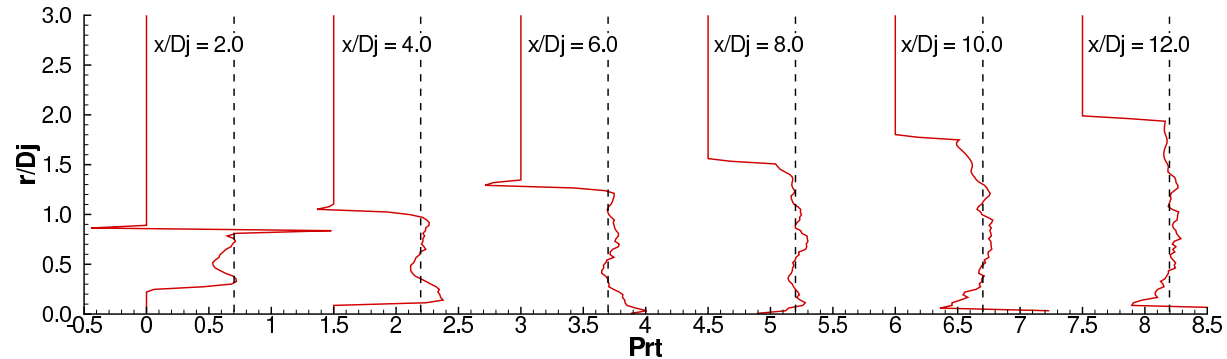
$$\epsilon_T = -\frac{\overline{\rho v'T'}}{\overline{\rho} \frac{\partial \overline{T}}{\partial y}}$$

$$Pr_t = \frac{\epsilon_m}{\epsilon_T}$$

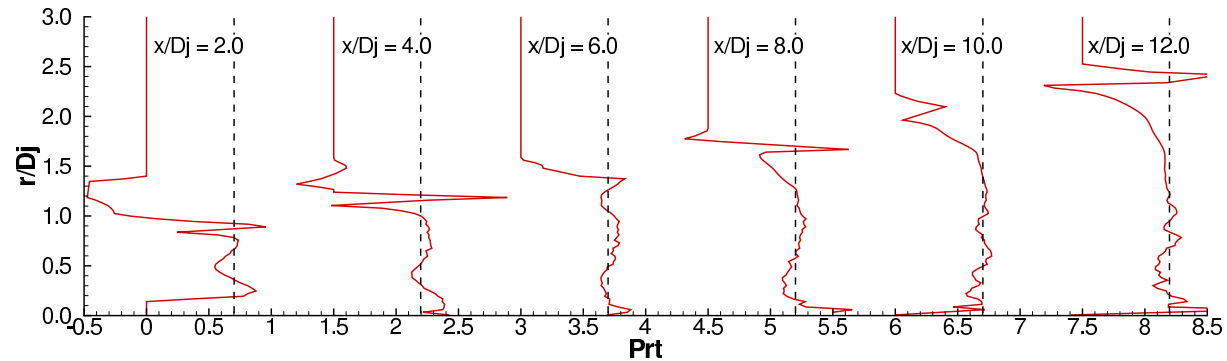
Radial Profiles – Pr_t



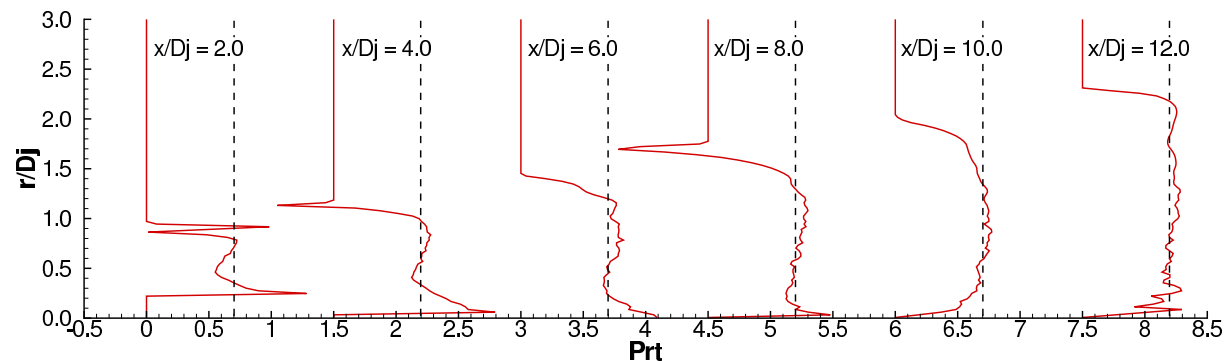
SP 3
 $M_a = 0.5$
 $T_j/T_\infty = 0.950$



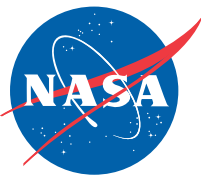
SP 23
 $M_a = 0.5$
 $T_j/T_\infty = 1.764$



SP 27
 $M_a = 0.9$
 $T_j/T_\infty = 1.764$



Summary and Conclusion



- LES and RANS methods were used to compute heated jet flows
 - RANS under-predicts spreading rate and inviscid core length (expected result)
 - LES agrees well with experimental data
- Turbulent heat flux
 - LES results consistent with literature
 - RANS model fails to replicate physics
 - Gradient diffusion assumption not appropriate for jets
- Turbulent Prandtl number
 - Little variation within the jet mixing layer
 - $Pr_t = 0.7$ is consistent with literature